

Simplify the following:

$$1. \frac{4x}{x^2-4} - \frac{2}{x+2}$$

$$\frac{4x}{(x-2)(x+2)} - \left(\frac{2}{(x+2)} \cdot \frac{(x-2)}{(x-2)} \right)$$

$$\frac{4x}{(x-2)(x+2)} - \frac{(2x-4)}{(x+2)(x-2)}$$

$$= \frac{4x-2x+4}{(x+2)(x-2)}$$

$$= \frac{2x+4}{(x+2)(x-2)}$$

$$= \frac{2(x+2)}{(x+2)(x-2)} = \frac{2}{x-2}$$

$$2. \frac{x^2}{x^2-1} + \frac{4x}{x^2-x}$$

$$\frac{x \cdot \frac{x^2}{(x+1)(x-1)}}{x} + \frac{4x \cdot \frac{(x+1)}{(x+1)}}{x(x-1)}$$

$$\frac{x^3 + 4x^2 + 4x}{x(x+1)(x-1)}$$

$$= \frac{x(x^2 + 4x + 4)}{x(x+1)(x-1)}$$

$$= \frac{(x+2)(x+2)}{(x+1)(x-1)}$$

$$3. \frac{3x}{x^2-9} + \frac{4}{2x-6} + \frac{5}{x+3}$$

$$\frac{2 \cdot \frac{3x}{(x+3)(x-3)}}{2} + \frac{4 \cdot \frac{(x+3)}{(x+3)}}{2(x-3)} + \frac{5 \cdot \frac{2(x-3)}{2(x-3)}}{2(x-3)}$$

$$\frac{6x + 4(x+3) + 5 \cdot 2(x-3)}{2(x+3)(x-3)}$$

$$\frac{6x + 4x + 12 + 10x - 30}{2(x+3)(x-3)}$$

$$= \frac{20x - 18}{2(x+3)(x-3)} = \frac{2(10x-9)}{2(x+3)(x-3)}$$

$$= \frac{10x-9}{(x+3)(x-3)}$$

$$4. \frac{x^2-2x}{x^2+2x+1} \div \frac{x^2+3x}{x^2+4x+3}$$

$$\frac{x(x-2)}{(x+1)(x+1)} \cdot \frac{(x+3)(x+1)}{x(x+3)}$$

$$= \frac{x-2}{x+1}$$

$$5. \frac{y^2-5y+4}{y^2-1} \div \frac{y^2-9}{y^2+5y+4}$$

$$\frac{(y-4)(y-1)}{(y+1)(y-1)} \div \frac{y^2-9}{y^2+5y+4}$$

$$\frac{(y-4)(y-1)}{(y+1)(y-1)} \cdot \frac{(y+4)(y+1)}{(y-3)(y+3)}$$

$$= \frac{(y-4)(y+4)}{(y-3)(y+3)}$$

$$6. \frac{\frac{1}{x-1} + x+3}{x-3 + \frac{1}{x+4}}$$

$$\left(\frac{1}{x-1} + \frac{(x+3)(x-1)}{(x-1)} \right) \div \left(\frac{(x-3)(x+4)}{(x+4)} + \frac{1}{x+4} \right)$$

$$\frac{1+(x+3)(x-1)}{x-1} \div \frac{(x-3)(x+4)+1}{x+4}$$

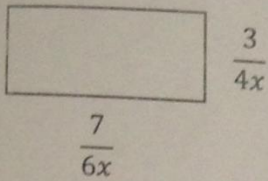
$$\frac{1+x^2+2x-3}{x-1} \div \frac{x^2+x-12+1}{x+4}$$

$$\frac{x^2+2x-2}{x-1} \div \frac{x^2+x-11}{x+4}$$

$$\frac{(x^2+2x-2)(x+4)}{(x-1)(x^2+x-11)}$$

$$\frac{x^3+6x^2+6x-8}{x^3-x-11x+11}$$

Find the perimeter and area of



Area:

$$\left(\frac{7}{6x}\right)\left(\frac{3}{4x}\right)$$

$$\frac{21}{24x^2} = \frac{7}{8x^2}$$

Perimeter:

$$\frac{3}{4x} + \frac{3}{4x} + \frac{7}{6x} + \frac{7}{6x}$$

$$3\left(\frac{6}{4x}\right) + \left(\frac{14}{6x}\right)\frac{2}{2}$$

$$\frac{18}{12x} + \frac{28}{12x}$$

$$\frac{46}{12x} = \frac{23}{6x}$$

➤ Goal 2: Asymptotes and Points of Discontinuity

8. Write a function that has the following characteristics: Hole at $x=4$ and a vertical asymptote at $x=-3$

$$\frac{(x-4)}{(x-4)(x+3)}$$

9) Determine the location of any points of discontinuity and describe them as either holes or asymptotes:

$$\begin{aligned} \text{a) } y &= \frac{x-5}{x^2-x-20} = \frac{\cancel{(x-5)}}{\cancel{(x-5)}(x+4)} \\ &= \frac{1}{x+4} \end{aligned}$$

Hole @ $x=5$
VA @ $x=-4$

$$\text{b) } y = \frac{x^2-x-6}{x^2-3x} ?$$

$$\frac{\cancel{(x-3)}(x+2)}{x\cancel{(x-3)}}$$

Hole @ $x=3$
VA @ $x=0$

10) What is the domain of the function in a)?

$$x \neq 5, -4$$

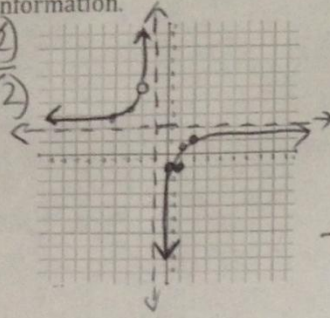
11. Graph the following function and find all of the requested information.

Factor numerator
by grouping!

$$f(x) = \frac{2x^2 + 3x - 2}{x^2 + 3x + 2}$$

$$\frac{(2x-1)(x+2)}{(x+1)(x+2)}$$

$$\frac{(2x^2 + 4x)(1x - 2)}{2x(x+2)(-1)(x+2)}$$



x	y
-4	3
-3	3.5
-2	hole @ 5
0	-1
1	1/2
2	1

Hole(s): $x = -2$ Vertical Asymptote(s): $x = -1$

Domain: $x \neq -2$ and -1

Horizontal Asymptote(s): $y = 2$

$$\frac{2(-2)-1}{-2+1} = \frac{-5}{-1} = 5$$

x-intercept(s): $(\frac{1}{2}, 0)$ y-intercept: $(0, -1)$ $y = \frac{2(0)-1}{0+1} = \frac{-1}{1} = -1$

$0 = 2x - 1$
 $\frac{1}{2} = \frac{2x}{2}$ $x = \frac{1}{2}$

➤ Goal 3: Solving Rational Equations

12. Solve for x & check for extraneous solutions: $\frac{2x+10}{2x+10} = \frac{6}{2x+10} \cdot \frac{4x-6}{4x-6}$

$$20x + 100 = 24x - 36$$

$$\underline{-20x} \quad \underline{+36} \quad \underline{-20x} \quad \underline{+36}$$

$$136 = 4x$$

$$x = 34$$

← does not make either denom = 0

13. Solve and check for extraneous solutions: $\frac{7}{x^2+3x-10} - \frac{3}{x+5} = \frac{4}{x-2}$

$$\frac{7}{(x+2)(x+5)} - \frac{3(x-2)}{(x+5)(x-2)} = \frac{4(x+5)}{(x-2)(x+5)}$$

$$7 - 3(x-2) = 4x + 20$$

$$7 - 3x + 6 = 4x + 20$$

$$13 - 3x = 4x + 20$$

$$\underline{-20} \quad \underline{+3x} \quad \underline{+3x} \quad \underline{-20}$$

$$\frac{-7}{7} = \frac{7x}{7}$$

$$x = -1$$

14. Solve for x and check for extraneous solutions: $\frac{x}{x+1} + \frac{5}{x} = \frac{18}{x}$

or use calc!

$$x = 3 \text{ and } 17.7$$

$$x^2 + 5 = 18x$$

$$\underline{-18x} \quad \underline{-18x}$$

$$x^2 - 18x + 5 = 0$$

$$\frac{18 \pm \sqrt{324 - 4(1)(5)}}{2(1)}$$

$$= \frac{18 \pm \sqrt{304}}{2}$$

$$= \frac{18 \pm 4\sqrt{19}}{2} = 9 \pm 2\sqrt{19}$$